Asymptotic Analysis of the Ratio of odd numbers in Pascal's Triangle

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Let us consider Pascal's Triangle mod 2; then, we will get Sierpiński's triangle. I will aim to calculate the ratio of ones in the triangle to the total number of numbers as the number of rows increases. Notice that the following functional equation holds.

$$f(2x) = \frac{3f(x)}{4} \tag{1}$$

Due to twice the number of rows, four copies of the previous iteration are added.

$$f(2^n x) = \left(\frac{3}{4}\right)^n f(x) \tag{2}$$

Given that $f(x) = c$, we get the following.

$$f(2^n) = \left(\frac{3}{4}\right)^n c \tag{3}$$

$$f(x) = \left(\frac{3}{4}\right)^{\log_2(x)} c = \frac{3^{\log_2(x)}}{4^{\log_2(x)}} c = \frac{3^{\log_2(x)}}{x^2} c = \frac{x^{\ln(3)/\ln(2)}}{x^2} c = x^{\ln(3)/\ln(2)-2} c \tag{4}$$

The last is asymptotic to $x^{\ln(3)/\ln(2)-2}$; through manual prediction, it seems the constant is around 1.71.